



THE WHITE ROSE GRID e-Science Centre

Searching for Mutually Unbiased Bases

Introduction

Mutually unbiased (MU) bases are an important, physically motivated tool allowing the reconstruction of quantum states with optimal efficiency. As of today, complete sets of MU bases have been constructed only for spaces of prime or prime power dimension. If the dimension is a composite number, 6,10,12, ... the existence of a complete set of MU bases has neither been proved nor disproved. In other words, it is unknown even for a 2x3 system whether there exists a set of observables which would realize optimal state reconstruction.

Prescribing a first Hadamard matrix, we construct all mutually unbiased to it, using an analytic approach based on the construction of Gröbner bases. We repeat this calculation many times, sampling all known complex Hadamard matrices, and never find more than two that are mutually unbiased. This result adds considerable support to the conjecture that no seven mutually unbiased bases exist in dimension six.

Use of the White Rose Grid

The code to sample nearly 6000

complex Hadamard matrices was written in Matlab, sampling all known complex Hadamard matrices, and required up to 16GB. This size and number of calculations cannot be practically run on desktop machines and the White Rose Grid was invaluable in providing the support necessary to complete this in a reasonable timeframe.

Results

No more than two mutually unbiased Hadamard matrices were found, adding considerable support to the conjecture that no seven mutually unbiased bases exist in dimension six. This result may have implications for quantum cryptography.

References

Stephen Brierley and Stefan Weigert, "Constructing Mutually Unbiased Bases in Dimension Six", Phys. Rev. A 79, 052316 (2009)

Further Information

Dr. Stefan Weigert
slow500@york.ac.uk
<http://maths.york.ac.uk>

